The Algorithms For Solving Task Of (r, p)-centroid On The Plane In L1-metric.

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Problem

It's a Stackelberg facility location game on Euclidian plane. Two players, called a leader and a follower, open facilities to service clients.

- Leader – '$\boldsymbol{p}$' facilities
- Follower – '$\boldsymbol{r}$' facilities

Each client chooses the closest facility.

Goal: maximize own market share.

ILP Formulation

$$y_k = \begin{cases} 1 & \text{follower opens facility inside } k \\ 0 & \text{otherwise.} \end{cases}$$

$$z_j = \begin{cases} 1 & \text{follower captures client } j. \\ 0 & \text{otherwise.} \end{cases}$$

Follower problem:

$$\max \sum_{j=1}^{n} w_j z_j$$

Subject to:

$$z_j \leq \sum_{k=1}^{n^2+n} a_{kj} y_j, j = 1, \dots, n, \qquad \sum_{k=1}^{n^2+n} y_k = r, y_k, z_j \in \{0,1\}$$

Current solution

We have the solution – a local search heuristic:

1. Leader places his facilities;
2. Follower replies;
3. Solve follower's problem **n** times for leader and follower;
4. Try to improve solution with local search.

What we do not like

1. Circles are hard to compute and operate with;
2. Finding all the intersections is also not nice problem to solve (NP-hard).
3. ~~Calculus and non-linear constraints~~

Our feelings

1. Helly's **Theorem** can be improved in case of R2 and squares
2. Clusterization can be done more efficiently with graph representation
3. Solving problem in L1-metric can make everyone a little happier
4. We can improve max. estimate number of clusters $n^2 + n$

What I’m doing

Developing an algorithm which can improve alternating heuristic of (r|p)-centroid problem on a plane and proving or disproving our guesses about the possible solution.

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Questions?