Context: Machine Learning for "Big Data"

- **Large-scale machine learning**: large N, large P
  - N: number of observations (inputs)
  - P: dimension of each observation
- **Regularized empirical risk minimization**: find x^* solution of

$$\min_{x \in \mathbb{R}^P} \frac{1}{N} \sum_{i=1}^{N} \ell(x^T a_i) \quad + \quad \lambda r(x)$$

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- Applications to any data-oriented field:
  - Vision, bioinformatics, speech, natural language, web.
- Main practical challenges:
  - Choosing regularizer r and data-fitting term ℓ.
  - Designing/learning good features a_i.
  - Efficiently solving the problem when N or P are very large.

This talk: Big-N Problems

- We want to minimize the sum of a finite set of smooth functions:

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- We are interested in cases where N is very large.
- We will focus on strongly-convex functions g.
- Simplest example is ℓ_2-regularized least-squares,

$$f_i(x) := (a_i^T x - b_i)^2 + \frac{\lambda}{2}\|x\|^2.$$

- Other examples include any ℓ_2-regularized convex loss:
  - logistic regression, Huber regression, smooth SVMs, CRFs, etc.

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  - Iteration cost is linear in N.
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- Stochastic gradient method [Robbins & Monro, 1951]:
  - Random selection of $i(t)$ from $\{1, 2, \ldots, N\}$,

$$x_{t+1} = x_t - \alpha_t f_{i(t)}'(x_t).$$

  - Iteration cost is independent of N.
  - Sublinear convergence rate: $O(1/t)$.

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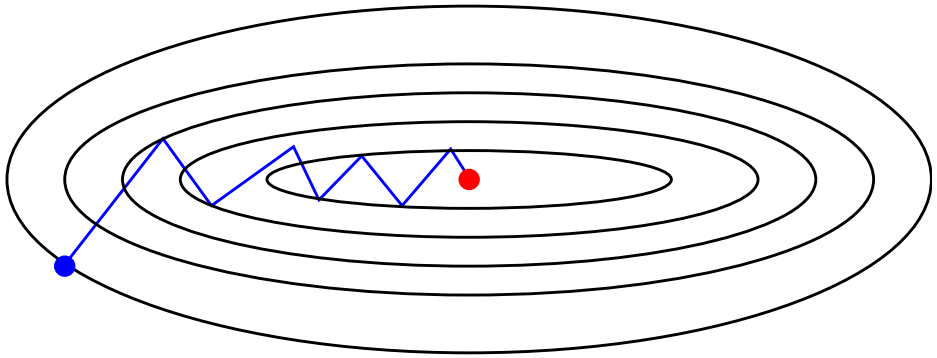

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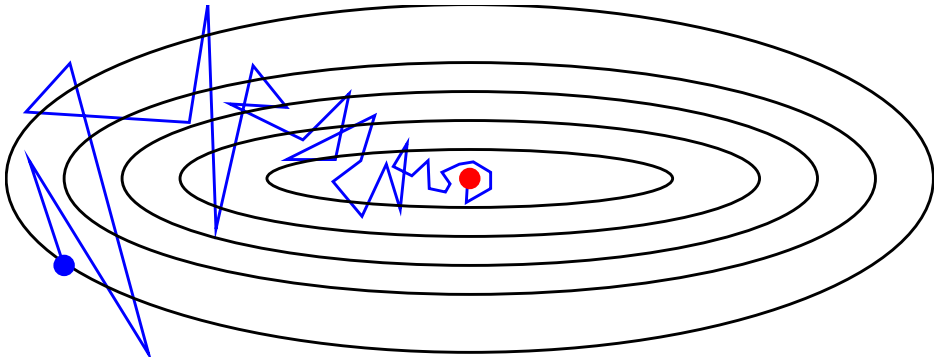
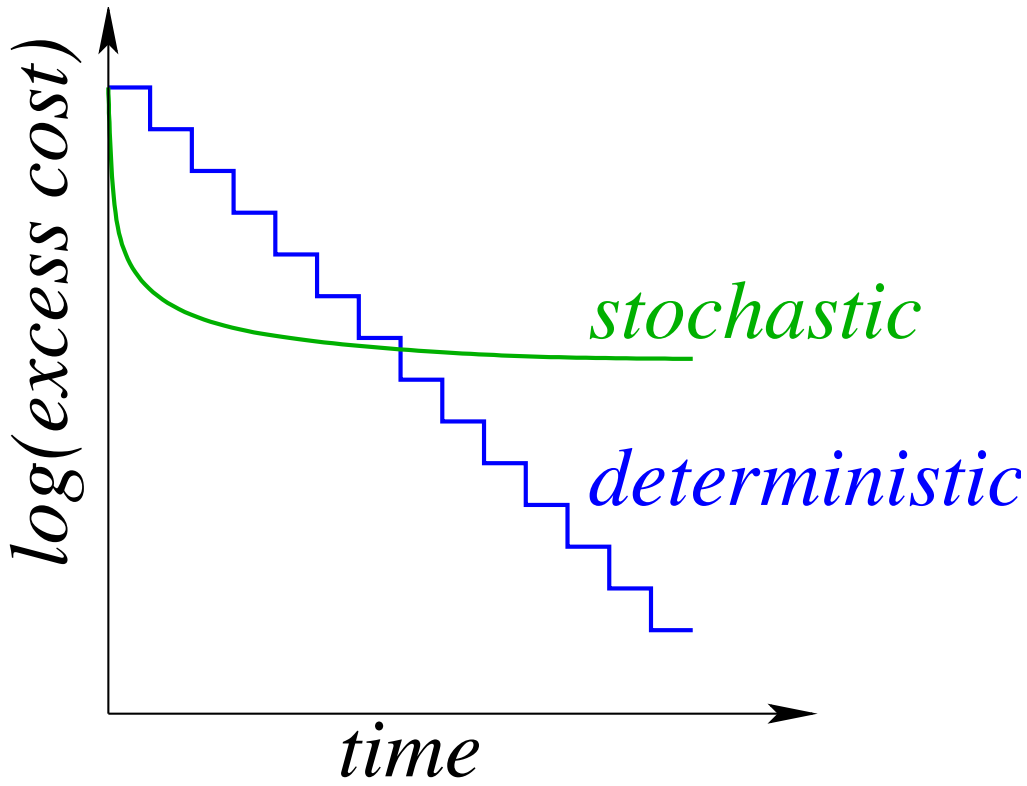


Motivation for New Methods

- FG method has $O(N)$ cost with $O(\rho^t)$ rate.
- SG method has $O(1)$ cost with $O(1/t)$ rate.



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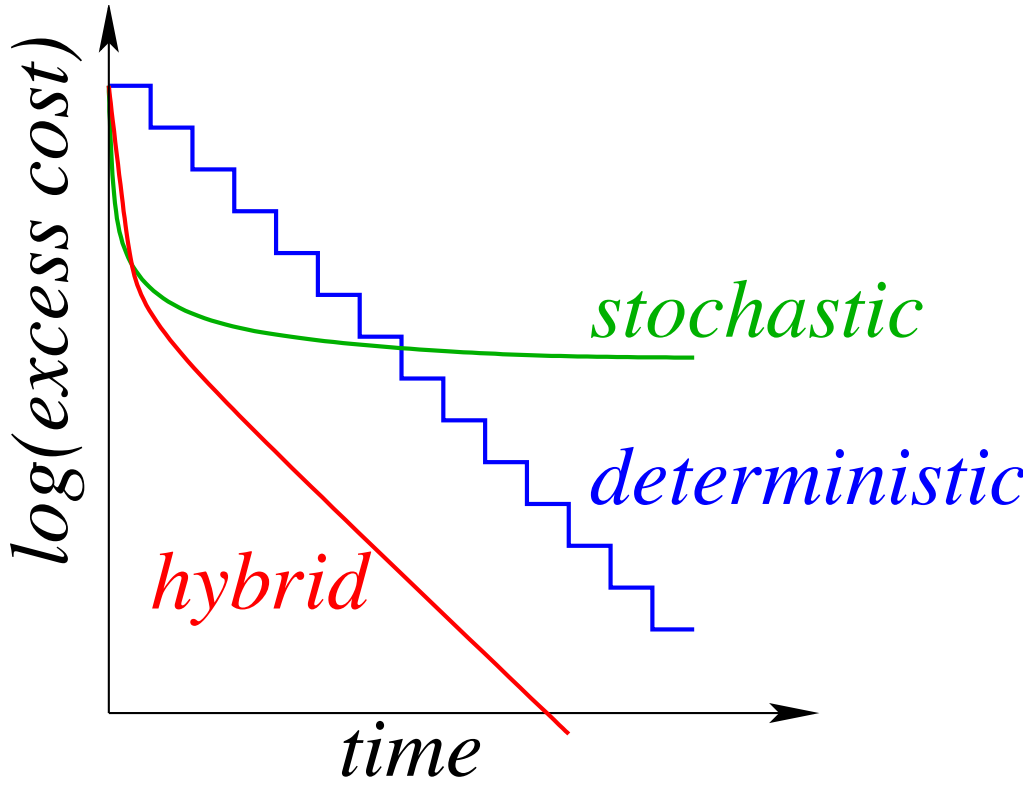

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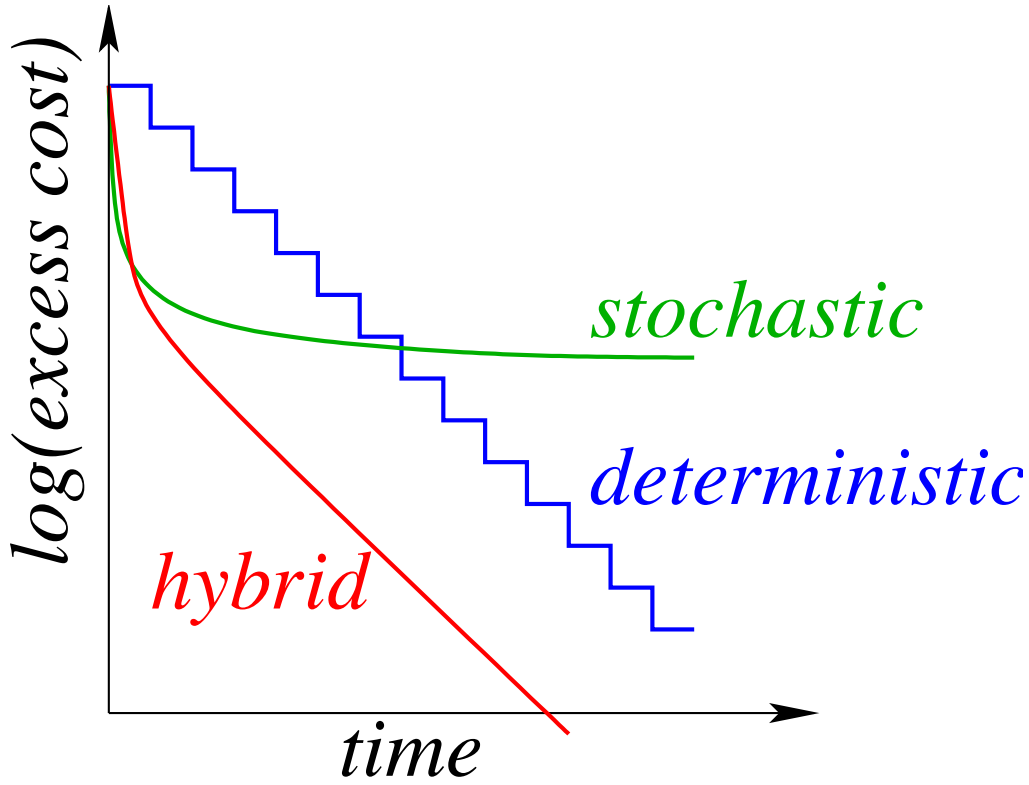

- Goal is $O(1)$ cost with $O(\rho^k)$ rate.

Prior Work on Speeding up SG Methods

A variety of methods have been proposed to speed up SG methods:

- **Step-size strategies, momentum, gradient/iterate averaging**
  - Polyak & Juditsky (1992), Tseng (1998), Kushner & Yin (2003) Nesterov (2009), Xiao (2010), Hazan & Kale (2011), Rakhlin et al. (2012)
- **Stochastic version of accelerated and Newton-like methods**
  - Bordes et al. (2009), Sunehag et al. (2009), Ghadimi and Lan (2010), Martens (2010), Xiao (2010), Duchi et al. (2011)

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- None of these methods improve on the $O(1/t)$ rate

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Existing linear convergence results:

- **Constant step-size SG, accelerated SG**
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  - Linear convergence up to a fixed tolerance: $O(\rho^t) + O(\alpha)$.
- **Hybrid methods, incremental average gradient**
  - Bertsekas (1997), Blatt et al. (2007), Friedlander and Schmidt (2012)
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- **Special Problems Classes**
  - Collins et al. (2008), Strohmer & Vershynin (2009), Schmidt and Le Roux (2012), Shalev-Shwartz and Zhang (2012)
  - Linear rate but limited choice for the f_i's

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  - f_i is convex, f_i' is $L-$continuous, g is μ-strongly convex.

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  - Assumes that gradients of other examples don't change.
  - This assumption becomes accurate as $||x^{t+1} - x^t|| \to 0$.
  - Stochastic variant of increment aggregated gradient (IAG). [Blatt et al. 2007]

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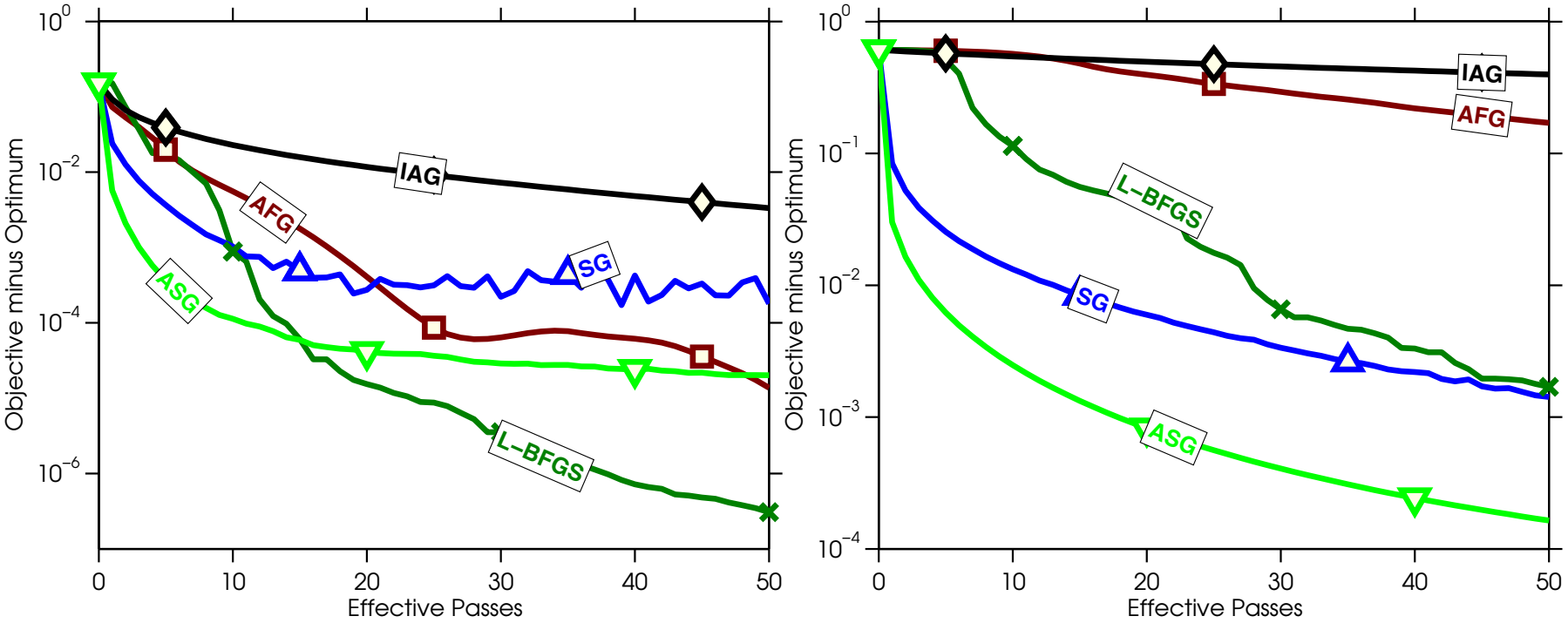
- Same algorithm could be used in non-convex case.
- Contrast with stochastic dual coordinate ascent:
  - Requires explicit strongly-convex regularizer.
  - Not adaptive to μ, does not allow $\mu = 0$.

Comparing FG and SG Methods

- quantum ($n = 50000$, $p = 78$) and rcv1 ($n = 697641$, $p = 47236$)



- Comparison of competitive deterministic and stochastic methods.

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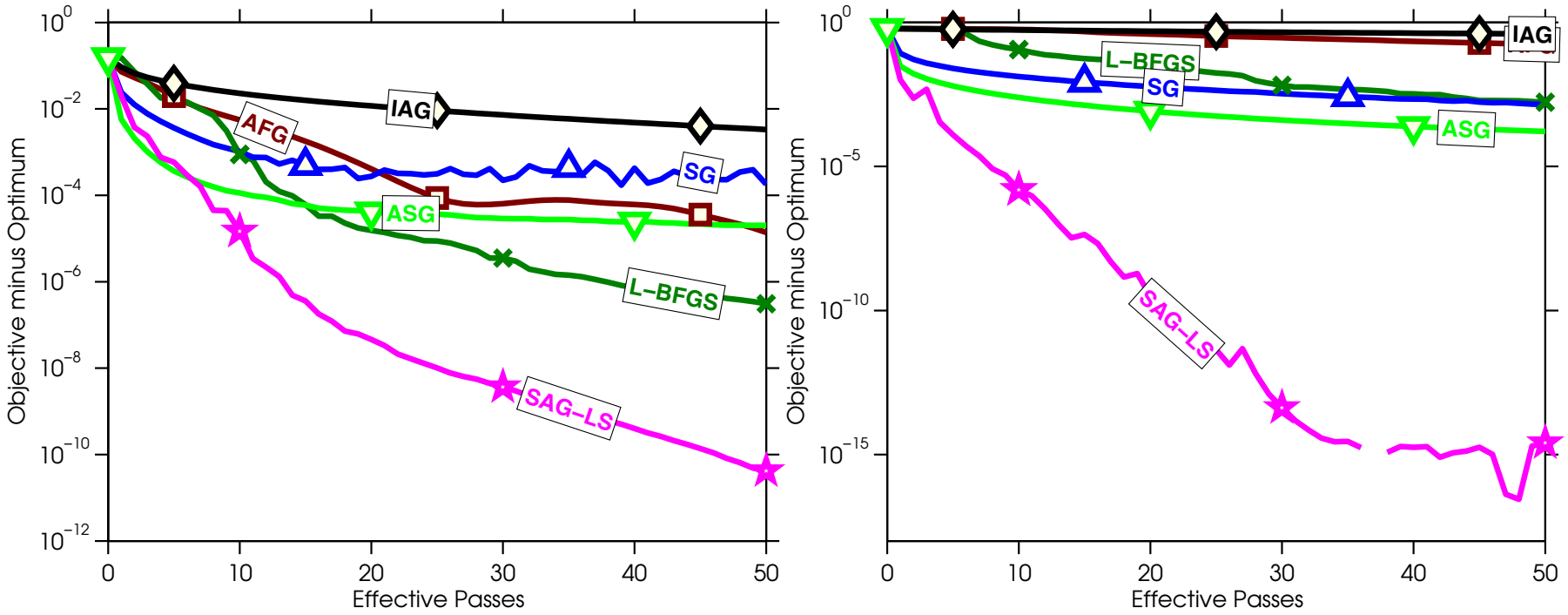

- SAG starts fast and stays fast.

SAG Compared to Coordinate-Based Methods

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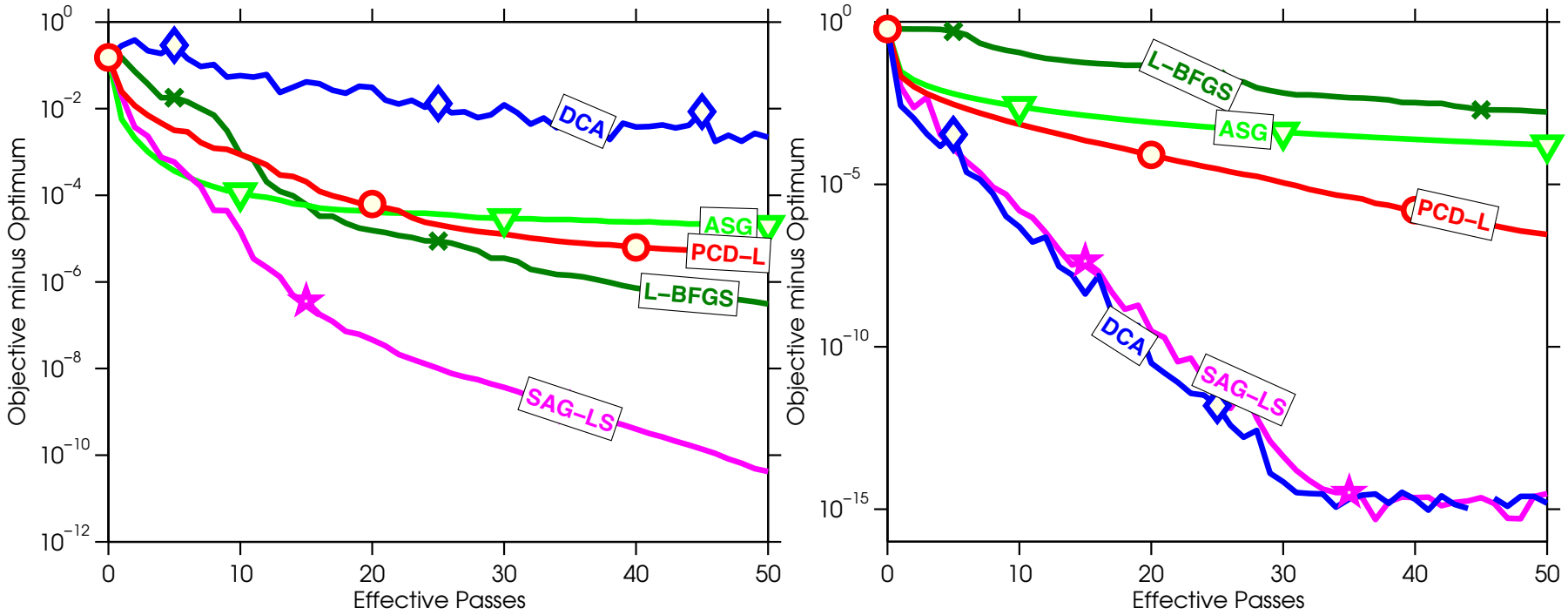

- PCD/DCA are similar on some problems, much worse on others.

SAG Implementation Issues

- while(1)
  - Sample i from $\{1, 2, \ldots, N\}$.
  - Compute $f_i'(x)$.
  - $d = d - y_i + f_i'(x)$.
  - $y_i = f_i'(x)$.
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  - Can we handle constraints or non-smooth problems?
  - Should we shuffle the data?

Implementation Issues: Normalization

- Should we normalize by N in the early iterations?
- The parameter update:
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Implementation Issues: Normalization

- Should we normalize by N in the early iterations?
- The parameter update:
  - $x = x - \frac{\alpha}{M} d$.
- We normalize by number of examples seen (M).
- Better performance on early iterations.
- Similar to doing one pass of SG.

Implementation Issues: Memory Requirements

- Can we reduce the memory?
- The memory update for $f_i(a_i^T x)$:
  - Compute $f_i'(a_i^T x)$.
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  - $y_i = f_i'(\delta)$.
- Use that $f_i'(a_i^T x) = a_i f_i'(\delta)$.
- Only store the scalars $f_i'(\delta)$.
- Reduces the memory from $O(NP)$ to $O(N)$.

Implementation Issues: Memory Requirements

- Can we reduce the memory in general?
- We can re-write the SAG iteration as:

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- The SVRG/mixedGrad method ('memory-free methods'):

$$x^{t+1} = x^t - \alpha\left(f_i'(x^t) - f_i'(\tilde{x}) + \frac{1}{N}\sum_{j=1}^{N} f_j'(\tilde{x})\right),$$

where we occasionally update $\tilde{x}$.

[Mahdavi et al., 2013, Johnson and Zhang, 2013, Zhang et al., 2013, Konecny and Richtarik, 2013, Xiao and Zhang, 2014]

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- Gets rid of memory for two f_i' evaluations per iteration.

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- Can we handle sparse data?
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- Apply previous k updates when it changes.

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- For sparse data, d_j is typically constant.
- Apply previous k updates when it changes.
- Reduces the iteration cost from $O(P)$ to $O(\|f_i'(x)\|_0)$.
- Standard tricks allow ℓ_2-regularization and ℓ_1-regularization.

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  - Increase L until we satisfy:

$$f_i(x - \frac{1}{L}f_i'(x)) \leq f_i(x) - \frac{1}{2L}\|f_i'(x)\|^2.$$

    (assuming $\|f_i'(x)\|^2 \geq \epsilon$)
  - Decrease L between iterations.

(Lipschitz approximation procedure from FISTA)

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  (Lipschitz approximation procedure from FISTA)
- For $f_i(a_i^T x)$, this costs $O(1)$ in N and P:

$$f_i(a_i^T x - \frac{f_i'(\delta)}{L}\|a_i\|^2).$$

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- If f_i are non-smooth, could smooth them or use dual methods.

[Nesterov, 2005, Lacoste-Julien et al., 2013, Shalev-Schwartz and Zhang, 2013]

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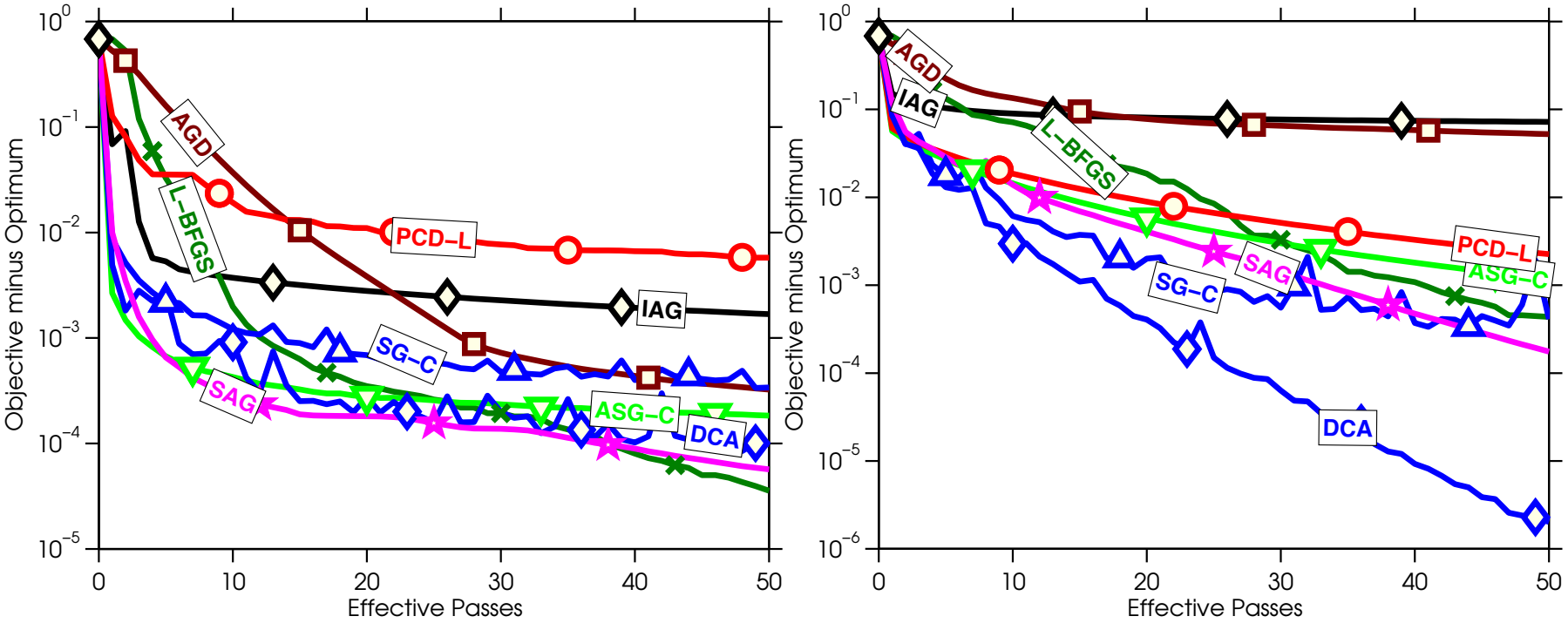
  - Combine with the line-search for adaptive sampling.

    (see paper/code for details)

SAG with Non-Uniform Sampling

- protein ($n = 145751$, $p = 74$) and sido ($n = 12678$, $p = 4932$)



- Datasets where SAG had the worst relative performance.

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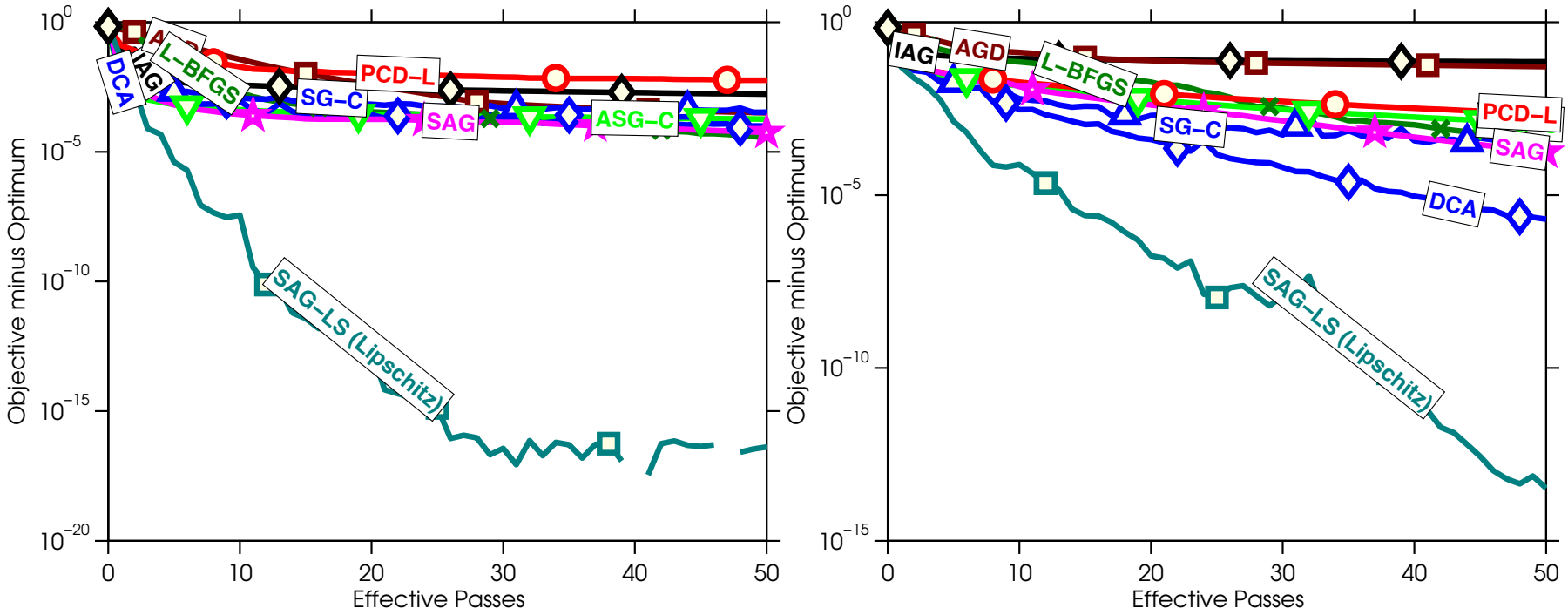

- Lipschitz sampling helps a lot.

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- Quasi-Newton method proposed that has empirically-faster convergence, but much overhead.

  [Sohl-Dickstein et al., 2014]

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- Future developments:
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  - Parallel/distributed methods.