Data-driven predictive modeling using FIML

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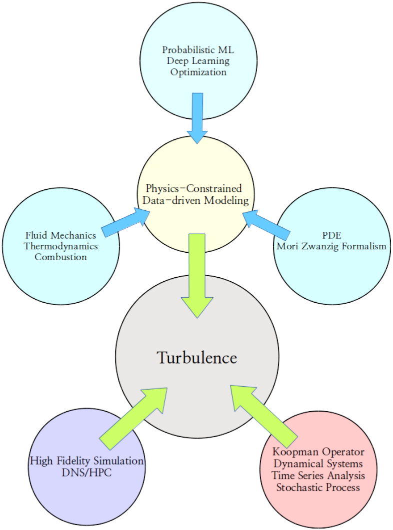
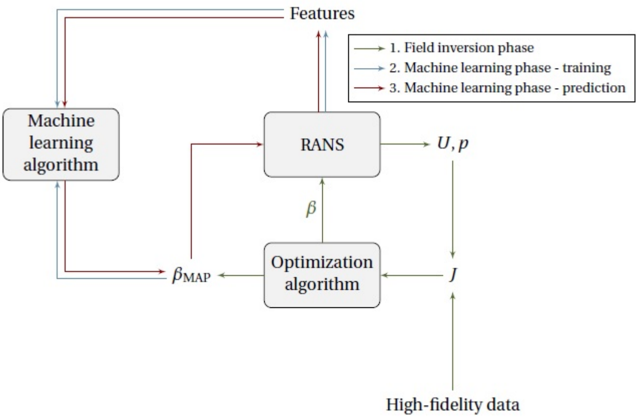


Introduction :



Introduction :

This research paper is based on theories of :

-Inverse problems

-Machine learning

-Turbulence

Research studies for using turbulence models and machine learning methods developed first by NASA projects.

Field inversion(=inverse problems) :

The goal of field inversion is to find the corrective field of which, when it is used in the RANS solver, the mean flow best approximates the high fidelity data.

$$h(\beta) = d_{true}$$

One approach is to use maximum likelihood principle by assuming data errors are independent and normally distributed

$$\beta_{true} = argmin||h(\beta) - d||^2$$

We can easily see it's a least square problem. Minimizing second norm of the difference between forward model output(solver data) and the data(sum of the d_{true})

We maximized the probability of receiving data given corrective field but there are some challenges !

Regularization :

- Forward model be non-linear then we may have several optimal solutions
- Our model might be incorrect or there be a lot of noise in the data(in this task low fidelity data obtained from solver
- There might be more than one corrective field
- There might be a small change in the data cause a large change in the resulting model.(=ill-conditioned problem)

We can overcome to the last two challenges by adding a regularization term to the objective function.(here we used Tikhonov Regularization)

$$\beta_{true} = argmin||h(\beta) - d||^2 + \alpha||h(\beta) - \beta_0||$$

Baysian approach is used and results distribution over corrective fields rather than a single corrective field.

$$\rho(\beta|d) \propto exp[(-0.5(d-h(\beta))^T C_m^{-1}(d-h(\beta)) - 0.5(\beta-\beta_{prior})^T C_\beta^{-1}(\beta-\beta_{prior})]$$

Parish and Duraisamy choose guassian with a constant diagonal for covariance matrix.

Optimization Techniques :

Gradient based optimization :
1.Objective function should be smooth
2.It conveges well when there is a single global minimum with appropriate steps

Genetic Algorithms :
1.Objective function be non-smooth
2.Multiple optimums
3.Good for discrete parameters

Kalman filter :
used for turbulence modelling but it requires a lot of simulations for high-dimensional problems.

In field inversion problems gradient based optimization perform better than others because it's effective in high number of parameters and we can obtain gradients of cost function independent from number of variables.

Sensitivity Analysis :

Calculation of the gradient of the objective function is computationally costly because it needs to optimize at each step so we need to choose the method carefully :

Three different methods are introduced in this paper :

1.Finite difference- expensive- has errors for small perturbations-only the output of a given input is needed for calculation(=black-box).

2.Complex variable method-it's needed to be set for complex arguments-it's not a black-box

3.Adjoint method-computational cost is independent from number of optimization variables-the most popular approach in field inversion.

Adjoint methodology :

In computation fluid dynamics tasks it is used to optimize lift-to-drag ratio, the pressure distribution, parameterizing geometry,etc

Continuous vs Discrete :

Discrete adjoint provides exact gradient continious adjoint approximates continious gradient.

Advantages of continious adjoint is that it is physically meaningful for the terms and the adjoint code is simpler and require less memory.

The implementation of continious adjoint is simpler in solvers like OpenFoam.

Turbulence :

It's part of fluid dynamics.

Till today there is no 100 percent accurate turbulent model.

We have three types of turbulence models :

DNS :

The most accurate and the most expensive considering computation.

LES :

Moderately accurate and also in costs.

RANS :

The least accurate and the least expensive model.

The idea of using machine learning and turbulence models is to improve the models that are less expensive by using machine learning algorithms.

Formulate the problem :

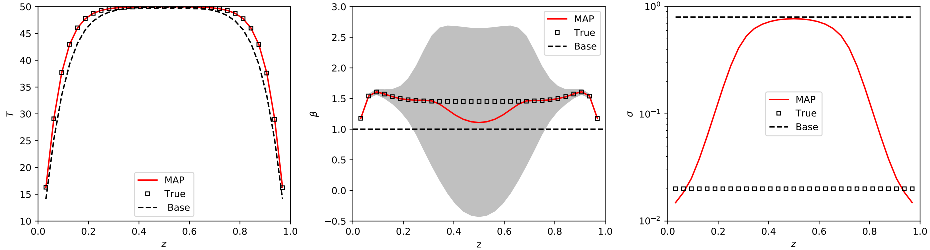
channel flow ate Re=2000 with high fidelity date(DNS) and Reynolds averaged data.

Gaussian process used as the machine learning algorithm.

Reproduction :
https ://github.com/afvk/FIML

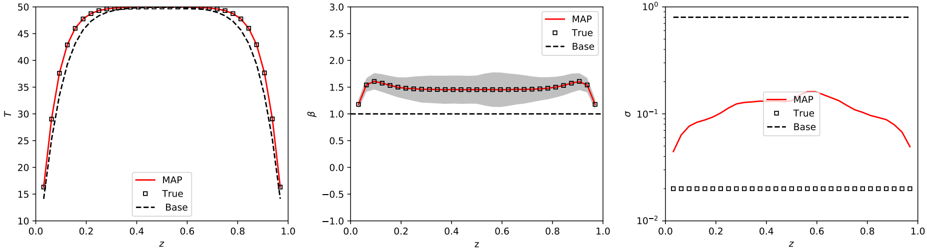
Results :

Diagonal covariance matrix with constant variance

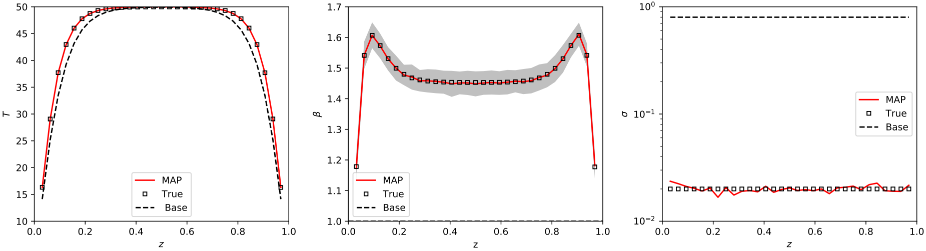


Results :

Diagonal covariance matrix with vector variance



Results :
Full covariance matrix



Results :

There are 5 different cases

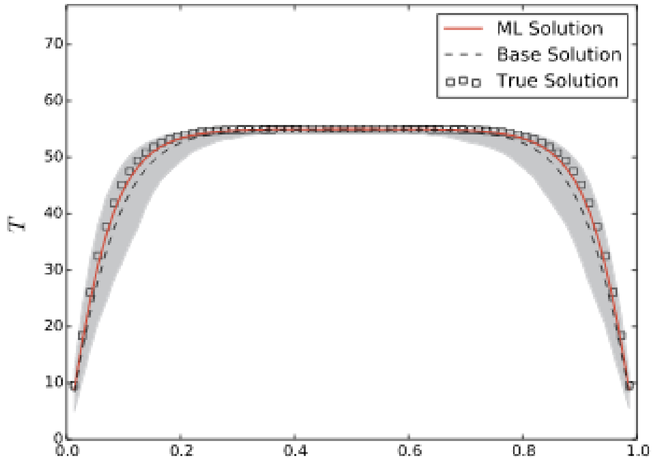
Table 2

Summary of cases used to test the predictive model. The L2 norm is used to compute the errors reported in column 5.

Case	Grid points	T_∞	Training data	$\lvert e_{map} - e_{model}\rvert / e_{model} \times 100$
1	71	28	$\mathbf{R} = \sigma^2 \mathbf{I}$	95.5%
2	71	55	$\mathbf{R} = \sigma^2 \mathbf{I}$	64.5%
3	71	$35 + 20\sin(2\pi z)$	$\mathbf{R} = \sigma^2 \mathbf{I}$	76.0%
4	71	$35 - 15z$	$\mathbf{R} = \sigma^2 \mathbf{I}$	98.3%
5	71	$15 + 5\cos(\pi z)$	$\mathbf{R} = \sigma^2 \mathbf{I}$	88.5%

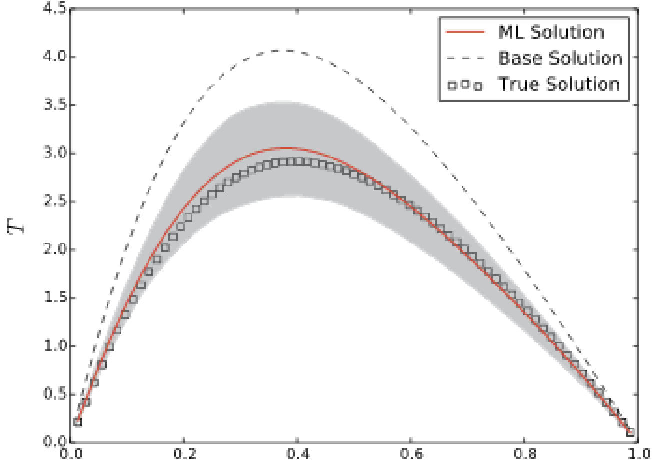
Results :

FIML results :



Results :

FIML results :



Future work and recommendations :

-There is one paper about FIML after this paper which gives better results than FIML classic.

"Field Inversion and Machine Learning With Embedded Neural Networks : Physics-Consistent Neural Network Training"

This paper is the result of phd thesis by Jonathan Holland.

-These papers used airfoil and simple channel flow. It can be a good idea to test it in more complicated geometries.

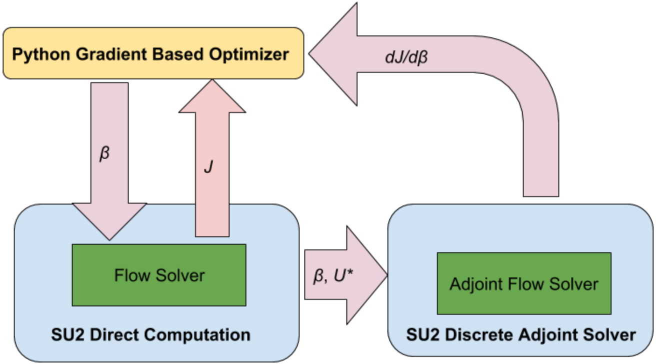
-Other machine learning methods can be tested.

-Till now only fully connected feed forward neural networks implemented. So, another possibility could be exploring other neural network architectures.

-Changing the features could be another option.

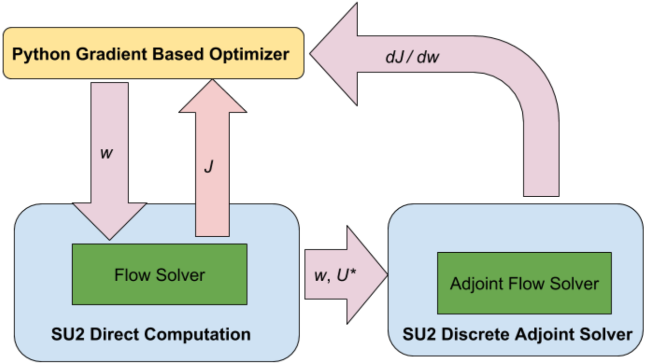
Conclusion :

-Classic FILML which is using machine learning and field inversion in two phase improved the model in some particular cases but FIML Direct architecture is better than Classic FIML.



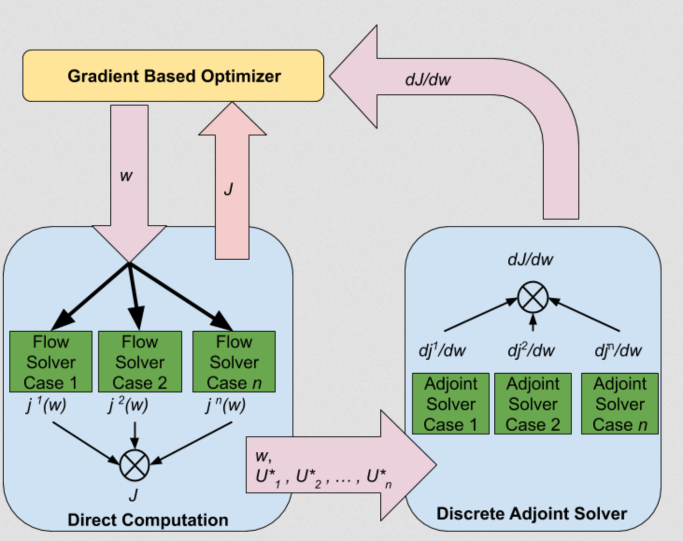
β	Turbulence Model Correction
U^*	Converged Flow Variables
J	Objective Function
$dJ/d\beta$	Gradient of Objective Function

Conclusion :



w	Neural Network Weights
*U**	Converged Flow Variables
J	Objective Function
dJ/dw	Gradient of Objective Function

Conclusion :



Gradient Based Optimizer
dJ/dw
w
J
Flow Solver Case 1
Flow Solver Case 2
Flow Solver Case n
j 1(w)
j 2(w)
j n(w)
J
Direct Computation
w,
U*1 , U*2 , ... , U*n
dJ/dw
dj1/dw
dj2/dw
djn/dw
Adjoint Solver Case 1
Adjoint Solver Case 2
Adjoint Solver Case n
Discrete Adjoint Solver

References :

-"A paradigm for data-driven predictive modeling using field inversion and machine learning" by Eric J. Parish, Karthik Duraisamy.
-Master thesis "Field inversion and machine learning in turbulence modeling" by A. F. van Korlaar.
-phd thsis "Integrated field inversion and machine learning with embedded neural network training for turbulence modeling." by Jonathan Holland.
-"Field Inversion and Machine Learning With Embedded Neural Networks : Physics-Consistent Neural Network Training" by Jonathan Holland,James Baeder, Karthik Duraisamy.

Thank you for your attention !